Exercise 18

Find the limit or show that it does not exist.

$$\lim_{x\to-\infty}\frac{4x^3+6x^2-2}{2x^3-4x+5}$$

Solution

Make the substitution, $u=-x$, so that as $x\to-\infty$, $u\to\infty$. Then multiply the numerator and denominator by the reciprocal of the highest power of u in the denominator.

$$\begin{aligned}
\lim_{x\to-\infty}\frac{4x^3+6x^2-2}{2x^3-4x+5} &= \lim_{u\to\infty}\frac{4(-u)^3+6(-u)^2-2}{2(-u)^3-4(-u)+5} \\
&= \lim_{u\to\infty}\frac{4(-u^3)+6u^2-2}{2(-u^3)+4u+5} \\
&= \lim_{u\to\infty}\frac{-4u^3+6u^2-2}{-2u^3+4u+5} \\
&= \lim_{u\to\infty}\frac{-4u^3+6u^2-2}{-2u^3+4u+5}\cdot\frac{\frac{1}{u^3}}{\frac{1}{u^3}} \\
&= \lim_{u\to\infty}\frac{(-4u^3+6u^2-2)\frac{1}{u^3}}{(-2u^3+4u+5)\frac{1}{u^3}} \\
&= \lim_{u\to\infty}\frac{-4+\frac{6}{u}-\frac{2}{u^3}}{-2+\frac{4}{u^2}+\frac{5}{u^3}} \\
&= \frac{-\lim\limits_{u\to\infty}4+\lim\limits_{u\to\infty}\dfrac{6}{u}-\lim\limits_{u\to\infty}\dfrac{2}{u^3}}{-\lim\limits_{u\to\infty}2+\lim\limits_{u\to\infty}\dfrac{4}{u^2}+\lim\limits_{u\to\infty}\dfrac{5}{u^3}} \\
&= \frac{-4+0-0}{-2+0+0} \\
&= 2
\end{aligned}$$